

TIDAL DISRUPTION FLARES: THE ACCRETION DISK PHASE

Matias Montesinos Armijo and José A. de Freitas Pacheco

*Université de Nice Sophia-Antipolis - Observatoire de la Côte d'Azur
Bd de l'Observatoire, BP 4229, 06304 Nice Cedex 4, France*

ABSTRACT

The evolution of an accretion disk, formed as a consequence of the disruption of a star by a black hole, is followed by solving numerically hydrodynamic equations. The present investigation aims to study the dependence of resulting light curves on dynamical and physical properties of such a transient disk during its existence. One of the main results derived from our simulations is that blackbody fits of X-ray data tend to overestimate the true mean disk temperature. In fact, the temperature derived from blackbody fits should be identified with the color X-ray temperature rather than the average value derived from the true temperature distribution along the disk. The time interval between the beginning of the circularization of the bound debris and the beginning of the accretion process by the black hole is determined by the viscous (or accretion) timescale, which also fixes the rising part of the resulting light curve. The luminosity peak coincides with the beginning of matter accretion by the black hole and the late evolution of the light curve depends on the evolution of the debris fallback rate. Peak bolometric luminosities are in the range $10^{45} - 10^{46}$ erg s $^{-1}$ whereas peak luminosities in soft X-rays (0.2-2.0 keV) are typically one order of magnitude lower. The typical timescale derived from our preferred models for the flare luminosity to decay by two orders of magnitude is about 3-4 years. Predicted soft X-ray light curves reproduce quite well data on galaxies in which a variable X-ray emission, possibly related to a tidal event was detected. In the case of NGC 3599 and IC 3599, data are well reproduced by models defined by a black hole with mass $\sim 10^7 M_{\odot}$ and a disrupted star of about one solar mass. The X-ray variation observed in XMMSL1 is consistent with a model defined by a black hole with mass $\sim 3 \times 10^6 M_{\odot}$ and a disrupted star of one solar mass, while that observed in the galaxy situated in the cluster A1689 is consistent with a model including a black hole of $\sim 10^7 M_{\odot}$ and a disrupted star of $\sim 0.5 M_{\odot}$.

Subject headings: Supermassive black holes; tidally disrupted stars; accretion disks

1. INTRODUCTION

Several flare events observed in surveys either in X-rays by *ROSAT* (Bade et al. 1996; Komossa & Bade 1999), *Chandra* and *XMM-Newton* (Donley et al. 2002; Esquej et al. 2007, 2008; Cappelluti et al. 2009, Maksym et al. 2010) or in the UV by *GALEX* (Gezari et al. 2006, 2009) are presently supposed to be associated with the tidal disruption of a star that has passed close to a super-massive black hole (SMBH), living in a “dormant” state in the core of the host galaxy. With the advent of several optical transient surveys such as *Pan-STARRS* (Chambers 2007), and the *Palomar Transient Factory* (Law et al. 2009), tidal flare candidates in the optical domain are also expected to be discovered. In fact, two possible candidates have recently been found in archival data of the Sloan Digital Sky Survey (SDSS; van Velzen et al. 2010). Flare properties are linked to the black hole mass and spin, making it an important tool for studying these objects, which otherwise undetectable due to the absence of any activity in their “quiet” phase. For SMBHs with masses lower than $\sim 10^8 M_\odot$, the tidal radius lies outside the horizon of a Schwarzschild black hole; this defines the upper limit of masses that can be probed from the analysis of the flare light curve.

Studies of the disruption of a star by the tidal field of an SMBH have been made since the late seventies (Frank 1978; Lacy, Townes & Hollenback 1982; Nolthenius & Katz 1982; Bicknell & Gingold 1983; Carter & Luminet 1983; Rees 1988; Ulmer 1999; Ayal, Livio & Piran 2000) and a general picture describing the sequence of events has now emerged. The physics of the disruption process is essentially described in terms of two dimensionless parameters: the “penetration” parameter $\lambda = R_t/R_p$, defined by the ratio between the tidal radius R_t and the periaapse distance R_p and the parameter η introduced by Press & Teukolsky (1977), defined by the square root of the ratio of the self gravity of the star to the tidal gravity at the surface when the star passes by the periaapse. If $\lambda \geq 5$, during the periaapse passage the star experiences a strong and rapid compression along directions perpendicular to the orbital plane and azimuthally, while stretching in the radial direction, leading to an increase of the central density by a factor $\sim \lambda^3$ (Carter & Luminet 1983). Using a “smoothed particle hydrodynamics” (SPH) code, Nolthenius & Katz (1982) investigated the tidal effects on a $1.0 M_\odot$ star, modeled by a polytrope of index $n = 1.5$, in a parabolic orbit around a $10^4 M_\odot$ black hole. According to their simulations, the star becomes marginally disrupted if $\eta \geq 1.6 - 5.8$. A similar analysis was performed by Khokhlov, Novikov & Pethick (1993), who used a three-dimensional Eulerian

hydrodynamical code. For a star described by a polytrope of index $n = 1.5$, they found that the dissipated tidal energy is comparable to the unperturbed binding energy if $\eta \sim 1.0 - 1.5$. For higher values of η the star is completely disrupted and otherwise partially stripped.

Early investigations developed a scenario in which the sudden tidal compression could lead to an increase in the thermonuclear energy generation rate, consequently increasing the luminosity of the star and eventually producing a supernova-like explosion (Carter & Luminet 1983; Pichon 1985). However, subsequent numerical simulations have shown that the attained compression ratio is much smaller and that the triple- α reaction is not ignited, excluding a possible detonation of the star (Bicknell & Gingold 1983). However, for a periaapse passage with $\lambda \sim 35$, significant processing of C,N,O can occur and the energy released by the thermonuclear reactions involving these elements could be comparable to the binding energy of the star, raising the possibility that the gas in the vicinity of the central BH may have a non-solar abundance pattern (Bicknell & Gingold 1983).

For a near parabolic orbit leading to a close encounter and to a tidal disruption of the star, there are two distinct phases in the dynamical evolution of the debris, in which an important amount of energy is released. The first is the “fallback” phase. The kinetic energy of the debris is much more important than its self-gravity and its internal energy. Nearly half of the debris is bound, initially forming gas streams with approximately ballistic trajectories, having a large spread in orbital periods and a mass distribution as a function of the energy nearly constant (Evans & Kochanek 1989; Ayal, Livio & Piran 2000). The most bound material returns to pericenter after a timescale t_{\min} which, in the majority of the cases of interest, is of the order of few years. The return of the bound debris initiates the formation of a small accretion disk around the BH. The circularization of the bound material is expected to be efficient since it is mainly driven by shocks between gas streams that converge at periaapse (Evans & Kochanek 1989; Laguna et al. 1993). The total energy radiated in this phase is of the order of $GM_{\text{bh}}\Delta m_*/2R_t$, where Δm_* is the mass of the bound material. For a BH of mass of $10^7 M_\odot$ and $\Delta m_* \sim 0.5 M_\odot$, this corresponds to an energy of about 2×10^{52} erg that, as we shall see later, represents about 10% of the total energy released in the later evolutionary period. The second, or “viscous”, phase initiates as soon as the disk begins to form, since the viscous forces responsible for the process of angular momentum transfer throughout the disk dissipate kinetic energy. By this mechanism the gas is heated and radiates; this is probably the major source of the flare luminosity.

The rate at which the bound debris returns to periastron and circularizes is often assumed to be identical to the rate at which the disk is formed or to the rate at which the BH accretes mass, radiating close or above the Eddington limit (Ulmer 1999). In fact, we will show that there is a time delay between the beginning of circularization and the instant the BH begins to accrete mass. During this short initial accretion phase, the BH accretes mass at a rate considerably higher than the fallback rate, and only in the final evolutionary stages are the rates of both processes comparable. A more detailed analysis of the disk properties was performed by Cannizzo, Lee & Goodman (1990), who solved a diffusion-like equation describing the evolution of the surface density. They concluded that the disk remains very luminous for several thousand years, representing a non negligible fraction of the time between successive stellar disruptions. Similar calculations were performed by Strubbe & Quataert (2009), who have also included in their model the reprocessing of the UV radiation by the unbound debris. As a consequence of such a process, a rich spectrum of very broad emission lines is produced.

More recently, Montesinos & de Freitas Pacheco (2011, hereafter MP11) studied the evolution of self gravitating disks around SMBHs, solving numerically the complete set of hydrodynamical equations. In the present paper, we report an investigation of the evolution of the accretion disk formed just after a tidal disruption event, using the code described in MP11. We will show that once the circularization process begins, two evolutionary phases can be distinguished. The first is an early and short phase in which the black hole is not accreting mass since the debris has not yet reached the last stable circular orbit, which occurs in a time fixed by the accretion or viscous timescale. In this phase, the flare begins to develop with the luminosity of the forming disk rising very rapidly. The second phase initiates once the black hole begins to accrete mass, which coincides with the instant of maximum flare luminosity. In its late evolution, the disk luminosity decays approximately at the fallback rate. The flare (and the disk) duration in the present models is typically of the order of a few up to tens of years, depending on the fallback rate. These values are considerably shorter than the timescales derived by Cannizzo, Lee & Goodman (1990). The paper is organized as follows: In Section 2 we discuss the model and equations; in Section 3 we present the main results and, finally, in Section 4 we give our conclusions.

2. THE DISK MODEL

Past investigations of the evolution of the accretion disk phase described the flow and the energy transfer by using several approximations. Moreover, most of these studies assumed that the disk is practically in place when its evolution begins. In fact, the accretion timescale $t_a \sim r/V_r$ differs from the fallback timescale of the bound debris, implying that there is a time lag between the beginning of the circularization of the bound material and the beginning of the accretion process by the central black hole. When the accretion process by the black hole begins, most of the bound material is not yet settled in orbit and, as we shall see, only in the late phases does the SMBH accrete mass at a rate comparable to the fallback rate.

The set of hydrodynamical equations (in cylindrical coordinates) describing the disk evolution as well as the details of the numerical code can be found in MP11 and here only some general aspects of the code are reviewed. The code is based on an eulerian formalism, using a finite difference method of second-order according to the Van Leer upwind algorithm on a staggered mesh. This means that scalar quantities such as surface density and scale of height are defined at the center of a cell, whereas vector quantities such as velocity and fluxes are defined at the interface between cells. Since disks considered in the present study are considerably smaller than those investigated by MP11, which have dimensions of ~ 50 pc, we have adopted an integration grid (one-dimensional) with 256 ring sectors instead of the original 1024 rings adopted by MP11. The inner radius of the grid coincides with the last stable circular orbit ($r_{\text{iso}} = 6GM_{\text{bh}}/c^2$) while the external radius is defined by the tidal radius. The time step is controlled by the Courant-Friedrich-Levy (CFL) condition, which states that the information cannot sweep a distance greater than the size of a cell. For further details, see MP11. Note that our axisymmetric geometry does not permit us to follow the debris evolution before circularization, when streams have eccentric orbits.

Other particular aspects should be mentioned. The first refers to the modeling of the viscosity responsible for the angular momentum transport, which in all past investigations was assumed to be described by the so-called “ α -model” introduced by Shakura & Sunyaev (1973). In this approach, the kinematic viscosity η (hereafter η represents the kinematic viscosity and not the Press-Teukolsky parameter, unless stated explicitly) due to the subsonic turbulence is given by

$$\eta = \alpha H c_s \quad (1)$$

where $\alpha \leq 1$ is a dimensionless coefficient, H is the ver-

tical scale of the disk, supposed to be of the same order as the typical (isotropic) turbulence scale ℓ_t and c_s is the sound velocity. As discussed in MP11, if the flow is self-regulated, it must be characterized by a critical Reynolds number \mathcal{R} determined by the viscosity above which the flow becomes unstable. Under these conditions, using the formalism developed by de Freitas Pacheco & Steiner (1976), the kinematic viscosity can be parametrized as

$$\eta = \frac{2\pi r V_\phi}{\mathcal{R}} \quad (2)$$

where r is the radial distance to the center of the disk and V_ϕ is the azimuthal velocity of the flow at that position. According to the analysis by Piran (1978), accretion disks modeled by such a viscosity prescription are thermally stable, while this is not the case for α -model disks.

The second aspect concerns the gravitational potential in the disk that, in the present case, is due essentially to the central BH since the disk self gravity is negligible. As in MP11, it was assumed here that the gravitational field of the BH is represented by the approximate potential of Paczynski-Wiita (Paczynski & Wiita 1980).

The third point concerns the modelization of the matter flux due to the fallback of the bound material. As mentioned above, the disk formed by the debris is supposed to have an extension comparable to the tidal radius, where we have assumed that an inward flux of matter is present. The timescale of the circularization process is still quite uncertain although some authors consider that it should be comparable to t_{\min} (Ulmer 1999). As we have mentioned previously, t_{\min} is of the order of a few years while the dynamical timescale at the level of the tidal radius is of the order of a day or even less (note that if we had adopted $\mu=1$ in Equation 5 below, the resulting timescale would be less than one year). Thus, we cannot exclude the possibility that the debris material circularizes in a timescale shorter than the accretion timescale and, in fact, here we have assumed that this occurs. Under these conditions, a modification in the outer boundary condition was introduced with respect to those discussed in MP11. In that work the boundary conditions were fixed by the following considerations: the (inward) mass flux at the inner radius (last stable orbit) corresponds to the mass accreted by the black hole and lost by the disk while at the outer radius the inward flux is taken to be zero, i.e., there is no source of matter for the disk. This is not the case in the present investigation since there is a continuous and variable source of matter due to the circularization process. Consequently, the original external boundary condition was modified and an inflow is now allowed such that the total rate of matter flowing into the disk is equal to the total rate at which

the debris returns to periaapse. According to early investigations by Rees (1988) and Evans & Kochanek (1989) the fallback rate is given by the relation

$$\frac{dM}{dt} \simeq \frac{1}{3} \frac{m_*}{t_{\min}} \left(\frac{t_{\min}}{t} \right)^{5/3} \quad (3)$$

where m_* is the mass of the disrupted star and

$$t_{\min} = \frac{\pi}{\sqrt{2}} \frac{R_p^3}{(GM_{\text{bh}} R_*^3)^{1/2}} \quad (4)$$

with R_* being the radius of the unperturbed star configuration. Here the periaapse distance will be taken to be equal to the tidal radius R_t , which was computed from the relation

$$R_t = \mu R_* \left(\frac{M_{\text{bh}}}{m_*} \right)^{1/3} \quad (5)$$

Most of authors in the literature take $\mu=1$ but here we will take $\mu \simeq 2.4$, corresponding approximately to the Roche model.

A more detailed analysis by Lodato, King & Pringle (2009) indicates deviations from Equation 3 at the early phases. According to them, only in the late evolutionary stages does the fallback rate vary as $t^{-5/3}$. As an attempt to include such deviations, we have adopted in our computations the following formula for the fallback rate

$$\frac{dM}{dt} = A \frac{(t/t_{\min})^{1/3}}{[a + (t/t_{\min})^2]} \quad (6)$$

where A is a normalization constant and t_{\min} is still given by Equation 4. Imposing that the time integral of Equation 6 be equal to $m_*/2$, which is the same condition satisfied by Equation 3, one obtains $A = \sqrt{3} a^{1/3} m_*/(2\pi t_{\min})$. The dimensionless parameter a permits control of the instant at which the maximum fallback rate occurs since this is given by $t_{\max} = \sqrt{a/5} t_{\min}$. Note that the late decay given by Equation 6 is proportional to $t^{-5/3}$ as in Equation 3. Equation 6 has also some computational advantages since the beginning of the circularization process can be taken at $t = 0$.

For technical reasons, we assume that the BH is initially surrounded by a very faint “phantom” disk, having a negligible mass in comparison with that of the bound debris. In fact, such a putative “phantom” disk is introduced only to avoid initially the presence of zeros in the integration grid; this does not affect the final results.

Two series of models were computed: those labeled A for which $a = 0.01$ and those labeled M for which $a = 1$. For M-models, the black hole always begins to accrete before the occurrence of the maximum of the fallback rate, while for A-models, depending on the adopted

critical Reynolds number, the black hole begins to accrete close to or after the occurrence of the maximum of the fallback rate. As we shall see below, the flare light curves derived from A and M series differ considerably, giving important information about the accretion timescale. Each computed disk model is characterized by the critical Reynolds number, the SMBH mass, the mass, and the radius of the disrupted star. These parameters for different computed models are given in Table 1.

3. RESULTS

3.1. The Disk Formation

Once the bound material begins to circularize (this corresponds to $t = 0$, the beginning of our integration procedure), the disk develops gradually and is formed in a time interval controlled essentially by the accretion timescale r/V_r , where r is equal to the tidal radius.

This gradual formation of the disk is illustrated in Figure 1, where several snapshots of the surface density profile derived from model A2 are shown. In the early evolutionary phases, the forming disk is characterized by a “torus-like” structure whose effective internal radius decreases as the material inspirals towards the black hole. The surface density of this structure increases in time as the front approaches the last stable orbit; This is a consequence of the fact that in these initial phases matter is deposited in the outer region of the disk at a rate higher than that which is conveyed inward. After reaching the last stable orbit (density profile labeled as “5” in Figure 1), the disk becomes more or less homogeneous and its late evolution is characterized by a decreasing surface density, which is a consequence of the exhaustion of the debris reservoir.

During the initial phases when the disk is still being formed and the black hole is not accreting mass, the flow of matter throughout the disk is not uniform. This can be seen in Figure 2 where snapshots of the mass flow profile for model A2 are also shown. The flow in the inner regions increases in amplitude as it propagates inwards and, when it reaches the last stable orbit (profile “5” in Figure 2), the accretion process by the black hole begins and a short, but significant peak in the luminosity curve is produced as we shall see below. Late in the process, the flow becomes practically uniform but with a decreasing amplitude, following the fallback rate of decay.

In Figure 3 the evolution of the accretion rate by the central black hole is plotted for models A1, A2, A3 and, for comparison, that of the fallback rate. Note that when the debris reaches the last stable orbit and the black hole begins to accrete mass, there is a sudden peak in the

Table 1: Model parameters: the first column identifies the model, the second gives the black hole mass, the third and the fourth give respectively the mass and radius of the disrupted star while the fifth gives the critical Reynolds number.

Model	M_{bh}/M_\odot	m_*/M_\odot	R_*/R_\odot	\mathcal{R}
A1, M1	1×10^7	1.0	1.0	500
A2, M2	1×10^7	1.0	1.0	1000
A3, M3	1×10^7	1.0	1.0	1500
A4, M4	1×10^7	2.0	1.7	1000
A5	1×10^6	1.0	1.0	500
A6, M6	3×10^6	1.0	1.0	500
A7, M7	3×10^6	2.0	1.7	500
A8, M8	3×10^6	2.0	1.7	1000
A9	3×10^6	1.0	1.0	1000
A10	3×10^6	1.0	1.0	100
A11	1×10^7	0.5	0.6	100
A12	1×10^7	0.5	0.6	1000

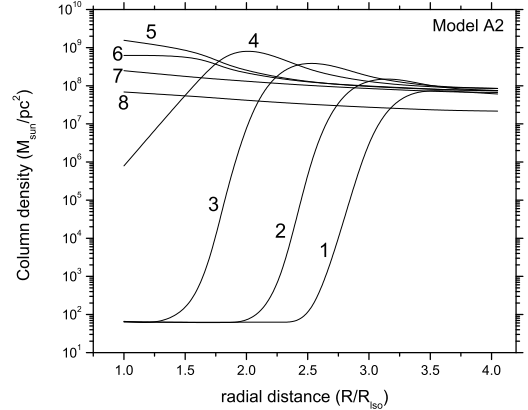


Fig. 1.— Different snapshots showing the evolution of the surface density. Distances are given in units of the last stable circular orbit. Labels correspond to the following instants of time: 1=0.0455 yr; 2=0.091 yr; 3=0.1823 yr; 4=0.273 yr; 5=0.4102 yr; 6=0.4558 yr; 7=0.501 yr; 8=1.367 yr. Profile 5 corresponds to the beginning of the accretion process by the black hole.

accretion rate which surpasses the fallback rate at that instant. Such a peak is in fact expected since its time integral corresponds approximately to the debris mass deposited up to that instant inside the disk. After the peak maximum, the accretion rate decays very rapidly and joins smoothly the fallback rate curve, initiating a late phase in which the accretion rate by the black hole is equal to the fallback rate. This corresponds to phases in which the matter flow through the disk is practically uniform and the luminosity is proportional to the accretion rate by the BH. Note that the peak in the accretion rate for models A1, A2, and A3 occurs at different times; the explanation for such a behavior is quite simple. These models have the same parameters but have flows characterized by different critical Reynolds numbers. Since the radial velocity scales as $V_r \propto V_\phi/\mathcal{R}$, the accretion timescale for these three models differs only in \mathcal{R} . Thus, one should expect that the ratio between instants of maximum accretion is equal to the ratio of the corresponding Reynolds number defining the model. The derived instants of maximum accretion for both series A and M are given in Table 2. For models A1, A2, and A3 the accretion peak occurs respectively at 0.205 yr, 0.415 yr, and 0.606 yr and their ratios follow quite well the ratios between 500, 1000 and 1500, the critical Reynolds number characterizing these models. From table 2, it is easy verify that these proportional ratios also hold for models M1, M2, and M3.

The accretion timescale controlling the disk formation scales as $t_a \propto \mathcal{R} \left(R_*^{3/2} m_*^{-1/3} \right)$, independent of the black hole mass. Models A1, A5, and A6 have the same parameters but differ in the black hole masses. As expected, the instant of maximum accretion derived for these models is nearly the same as that which can be verified by simple inspection of Table 2. A further check can be performed by plotting the derived instant of maximum accretion t_p as a function of t_a expressed in terms of the critical Reynolds number and parameters of the disrupted star. This was done in Figure 4 for models of series “A”. As expected, the correlation is highly significant (correlation coefficient 99.6%), providing a further test for the adequacy of the algorithm adopted for the numerical solution of the hydrodynamic equations. The solid line in the plot corresponds to the relation

$$\log t_p = -3.381 + \log \mathcal{R} \frac{R_*^{3/2}}{m_*^{1/2}} \quad (7)$$

3.2. The Light Curve

The shape of the light curve depends strongly on the ratio between the accretion time t_a and the instant of

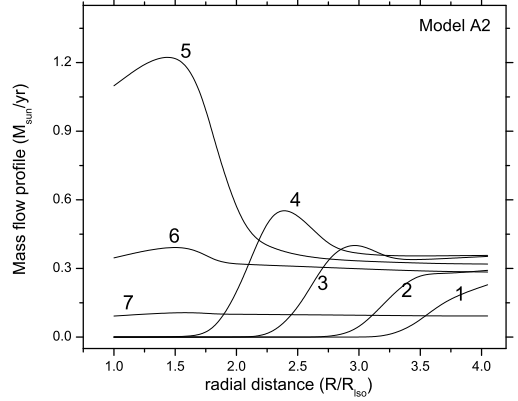


Fig. 2.— Snapshots showing the evolution of the mass flow. Distances are given in units of the last stable circular orbit. Labels correspond to the following instants of time: 1=0.0455 yr; 2=0.091 yr; 3=0.1823 yr; 4=0.273 yr; 5=0.4102 yr; 6=0.4558 yr; 7=0.501 yr. Profile 5 corresponds to the beginning of the accretion process by the black hole.

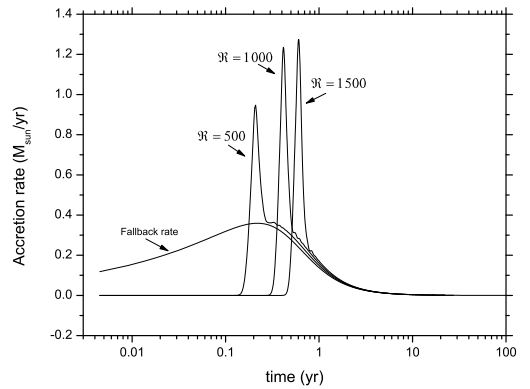


Fig. 3.— The evolution of the accretion rate by the black hole for models differing only in the critical Reynolds number. The evolution of the fallback rate for $a = 0.01$ is also shown.

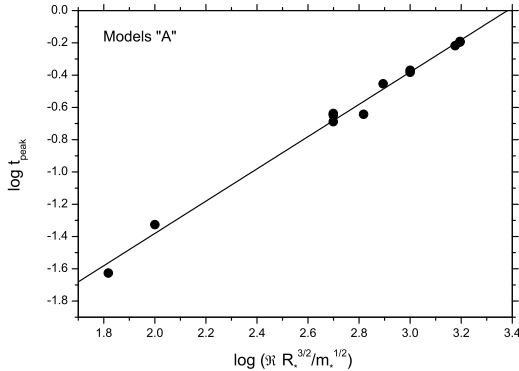


Fig. 4.— Instant of the peak in the accretion rate versus the accretion timescale. Filled dots correspond to values derived from “A” models and solid line to the relation given in the text.

Table 2: Disk and flare properties: the first column identifies the model, the second gives the ratio between the tidal and the last stable orbit radii, a measure of the disk extension. The third column gives t_{min} in years and the last two columns give the instant of the maximum accretion in years respectively for models of series “A” and “M”.

Model	R_t/R_{lso}	t_{min}	$t_p(A)$	$t_p(M)$
1	4.05	4.905	0.205	0.228
2	4.05	4.905	0.415	0.474
3	4.05	4.905	0.606	0.684
4	5.47	5.436	0.638	0.410
5	18.84	1.551	0.231	-
6	9.06	2.686	0.225	0.315
7	12.22	2.977	0.352	0.498
8	12.22	2.977	0.644	0.779
9	9.06	2.686	0.427	-
10	9.06	2.686	0.047	-
11	3.07	4.559	0.024	-
12	3.07	4.559	0.228	-

maximum fallback rate t_{max} . If $t_a/t_{max} < 1$, i.e., the material reaches the last stable orbit before the occurrence of the maximum fallback rate (all M models satisfy this condition), a “bump” in the light curve appears just after the luminosity peak. This feature can be explained in the following way: The luminosity maximum coincides with the peak in the accretion rate. After the peak, the BH accretion rate has a fast decay and gradually joins the fallback rate curve. Since the maximum rate of the fallback material has not yet occurred, the accretion rate will pass by the secondary maximum, producing the aforementioned “bump”. In the opposite case ($t_a/t_{max} > 1$), satisfied by most of the “A” models, such a “bump” does not appear because the black hole begins to accrete after the occurrence of the maximum in the fallback rate. In this case, after the peak, the luminosity decays as $t^{-5/3}$ similar to the late evolution of models “M”. This behavior is shown in Figure 5 where the bolometric light curves for models A4 and M4 are shown for comparison. Such a trend differs from that derived from the similarity solution obtained by Cannizzo, Lee & Goodman (1990), which indicates a late luminosity decay as $t^{-19/16}$.

The characteristic timescale for the luminosity to decay by two orders of magnitude after the peak is also different for models of series “A” and “M”. The former have on average typical decay timescales of about 3.2 yr, while for the later values are on average around 43.3 yr. The narrow peak feature is also shorter for “A” models (~ 5.5 months against ~ 8.3 months on the average for M models). Moreover, “A” models generally produce flares with peak luminosities of the order of few 10^{46} erg s $^{-1}$ while the peak luminosity of models “M” is, on average, one order of magnitude lower. As expected, peak bolometric luminosities L_p correlate quite well with the peak accretion rate \dot{R}_p by the central black hole. For models “A”, a best fit of derived data gives

$$\log L_p = (46.209 \pm 0.031) + (1.060 \pm 0.110) \log \dot{R}_p \pm 0.106 \quad (8)$$

with a correlation coefficient of 95.0%. In this relation the peak luminosity is in erg s $^{-1}$ and the peak accretion rate is in $M_\odot \text{yr}^{-1}$. The total energy radiated by the event is proportional to the total accreted mass by the black hole ($E \sim \Delta m_* c^2$), being of the order of few 10^{53} erg.

It should be emphasized again that during the rising phase of the flare, the black hole is not accreting mass. The radiation comes from the material heated by viscous dissipation in a disk still in formation and far from equilibrium. When the accretion process begins, the resulting luminosity peak exceeds the Eddington value. How-

ever, one may wonder whether the supercritical Eddington regime should be applied to a disk geometry (see, for instance, Heinzeller & Duschl 2007). In the case of a disk, the condition for having equilibrium along the z -axis is local. Near the last stable orbit, tidal forces due to the SMBH and pressure gradients balance each other and the condition for stability, i.e., the condition for radiation pressure gradients to not overtake gravity is (MP11)

$$Q_{\text{rad}}(r) < \frac{cGM_{\text{bh}}}{2r^2} \frac{\Sigma}{\tau_{\text{ef}}} \left(\frac{H}{r} \right) \quad (9)$$

where Q_{rad} is the radiative flux, Σ is the surface density, τ_{ef} is the effective optical depth and H is the disk scale of height. The present models satisfy this condition for the following reason: As the radiation pressure begins to inflate the disk, the fraction of advected energy increases, permitting higher accretion rates without increasing the radiative flux. It should be emphasized that this cooling effect is strengthened by the photon-trapping mechanism, as first pointed out by Begelman (1978) and Oshuga et al. (2002).

3.3. Physical Properties of the Disk

The evolution of the surface density is similar for all computed models as discussed previously. In this section, the evolution of the disk temperature will be examined. The local effective temperature of the disk was derived by equating the energy rate per unit of area dissipated by viscous forces, corrected by advection and photon-trapping, to the energy rate per unit of area radiated away (see MP11 for details). Inside the disk, photons interact with matter essentially by scattering on free electrons and by free-free processes. The effective optical depth of the photon along the z -axis is $\tau_{\text{ef}} = \sqrt{3\tau_{\text{ff}}(\tau_{\text{ff}} + \tau_{\text{s}})}$, where τ_{ff} is the optical depth due to free-free processes and τ_{s} is the Thomson scattering optical depth. Figure 6 shows snapshots of the evolution of the effective optical depth profile for model A2. Note that either the evolving “torus”-like structure or the disk when formed (profile labeled “5”) is always optically thick, but the integrated disk spectrum is not that of a pure blackbody for two main reasons: firstly the effective temperature varies across the disk surface and secondly, on the average, $\tau_{\text{s}}/\tau_{\text{ff}} \sim 1$ throughout the disk. Deviations from a blackbody spectrum are more severe when $\tau_{\text{s}}/\tau_{\text{ff}} \gg 1$ since in this case the local flux is approximately given by $F(\nu) \propto B_{\nu}(T)\sqrt{\tau_{\text{ff}}(\nu)/\tau_{\text{s}}}$.

The evolution of the effective temperature profile for model A2 is shown in Figure 7. Since the dissipated energy rate per unit of area due to viscosity is proportional

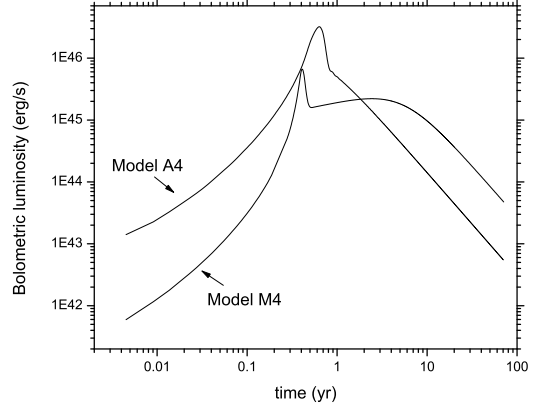


Fig. 5.— Flare light curves for two models having the same parameters but with a different evolution in the fallback rate. Notice the presence of a “bump” just after the peak luminosity in the light curve of model M4.

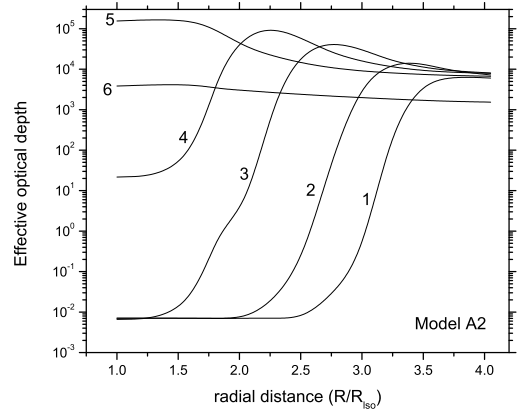


Fig. 6.— Evolution of the effective optical depth profile for model A2. Distances are in units of the last stable circular orbit. Labels correspond to instants: 1=0.0455 yr; 2=0.091 yr; 3=0.1823 yr; 4=0.273 yr; 5=0.4102 yr; 6=1.367 yr.

to the surface density, the evolution of the effective temperature profile follows the former. In the early phases, profiles indicate that the temperature increases inwards and in time as the material approaches the last stable orbit. The maximum value is attained in the inner region of the disk at the instant when the black hole begins to accrete mass (see profile “5” in Figure 7). All models show the same trend but the temperature values depend on the adopted parameter, in particular that of the black hole mass.

Observers fit, in general, the spectral data of flare candidates by an equivalent blackbody distribution characterized by a unique temperature. As seen in Figure 7, the effective temperature in the early evolutionary phases varies considerably along the disk surface and only in the late evolutionary stages does it become more uniform. In order to compare temperatures derived from our models with some real data, we have defined a suitable mean effective temperature for the disk by the relation $\langle T \rangle \equiv \langle T^4 \rangle^{1/4}$ where

$$\langle T^4 \rangle = \frac{\int_{R_{\text{iso}}}^{R_t} r T^4(r) dr}{\int_{R_{\text{iso}}}^{R_t} r dr} \quad (10)$$

The evolution of such a mean temperature for models A1, A5, A6 and A3 is shown in Figure 8. Models A1, A5 and A6 have the same parameters but differ on the BH mass. Despite the fact that in the first 2 months of the flare evolution, when the disk is still being formed, there is no difference among these three models, the maximum attained temperature and the late evolution differ in the sense that models with more massive black holes produce “colder” disks, since the dissipation rate due to viscous forces is less important. In fact, our models indicate that $\langle T \rangle_{\text{peak}} \propto M_{\text{bh}}^{-5/2}$. For steady disks whose viscosity is given by the “ α -model”, it is well known that the disk temperature varies as $T \propto M_{\text{bh}}^{-1/4}$. Our models have a steeper dependence, probably due to a different temperature profile resulting from our viscosity prescription and non-steady conditions. Model A3 should be compared with model A1, as both have BHs and disrupted stars of the same mass but differ in the critical Reynolds number. This affects the instant of maximum temperature and the late behavior of the temperature decay but not the maximum temperature attained at the peak luminosity.

Snapshots of the spectral evolution of the radiation emitted by the disk derived from model A3 are shown in Figure 9. The dashed curve corresponds to the “true” spectrum, which was calculated taking into account the distribution of the effective temperature along the disk surface, while the solid curve represents the spectrum de-

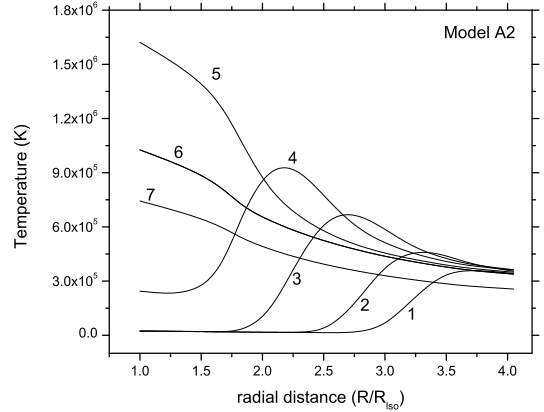


Fig. 7.— Evolution of the effective temperature profile for model A2. Distances are in units of the last stable circular orbit. Labels correspond to instants: 1=0.0455 yr; 2=0.091 yr; 3=0.1823 yr; 4=0.273 yr; 5=0.4102 yr; 6=0.4558 yr; 7=1.367

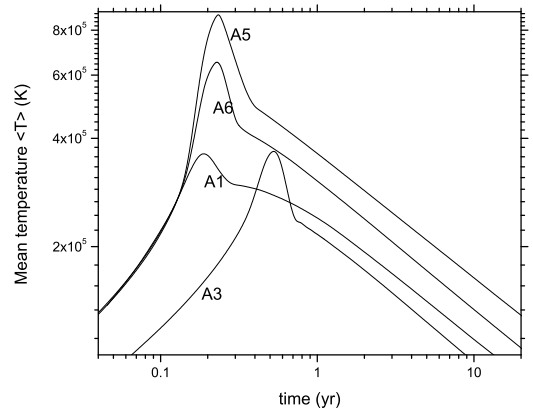


Fig. 8.— Evolution of the average effective temperature of the disk for different models.

rived from a blackbody distribution characterized by the mean effective temperature as previously defined. The upper panel ($t = 0.09$ yr) shows the spectrum when the disk is still in formation while the first middle panel ($t = 0.60$ yr) shows the spectrum at the luminosity peak. The second middle panel ($t = 9.11$ yr) and the bottom panel ($t = 18.23$ yr) show spectra in late evolutionary phases of the disk (or of the flare). Note that as the disk evolves the mean effective temperature increases, as does the emission at short wavelengths, whereas in the late stages the opposite behavior is seen. “Real” and blackbody spectra agree in the Rayleigh-Jeans region but differ considerably in the Wien’s region where the blackbody approximation underestimates the true flux. In particular, the blackbody approximation underestimates the true X-ray emission from the disk. As a consequence, models using blackbody distribution to describe the observed X-ray emission, require temperatures higher than the mean effective values derived from models taking into account the temperature variation along the disk surface. We must have this fact in mind when analyzing some flare data in the next section.

3.4. Comparison with Observations

Firstly it should be mentioned that the aim of this work was to investigate the evolution of the disk originated from the debris of a disrupted star by an SMBH and how the different parameters affect the properties of the light curve of the resulting outburst. No particular effort was made to fit the observed parameters of different tidal flare candidates existing in the literature.

3.4.1. NGC 3599

NGC 3599 is a non-active S0 galaxy ($z = 0.00277$) in which a variable X-ray emission was detected and whose origin could be due to a tidal disruption event (Esquej et al. 2007, 2008). The source was detected originally by the *XMM-Newton* slew survey in 2003.89 and was further observed by this telescope in 2006.49, by *Swift/X-Ray Telescope* in 2006.94 and by *Chandra* in 2008.1. Figure 10 (upper panel) shows the derived X-ray light curve (0.2–2.0 keV range) for model A2 and the X-ray luminosities derived from the aforementioned observations. Note that according to our model, the soft X-ray peak luminosity attained a value of 1.8×10^{44} erg s $^{-1}$ and occurred about 1.5 yr before detection.

When the source was first detected, the disk average effective temperature derived from model A2, was is $\langle kT \rangle = 0.016$ keV while the analysis of *XMM-Newton* data by Esquej et al. (2008) indicates values in the range 0.04 – 0.10 keV. These values are higher respectively by

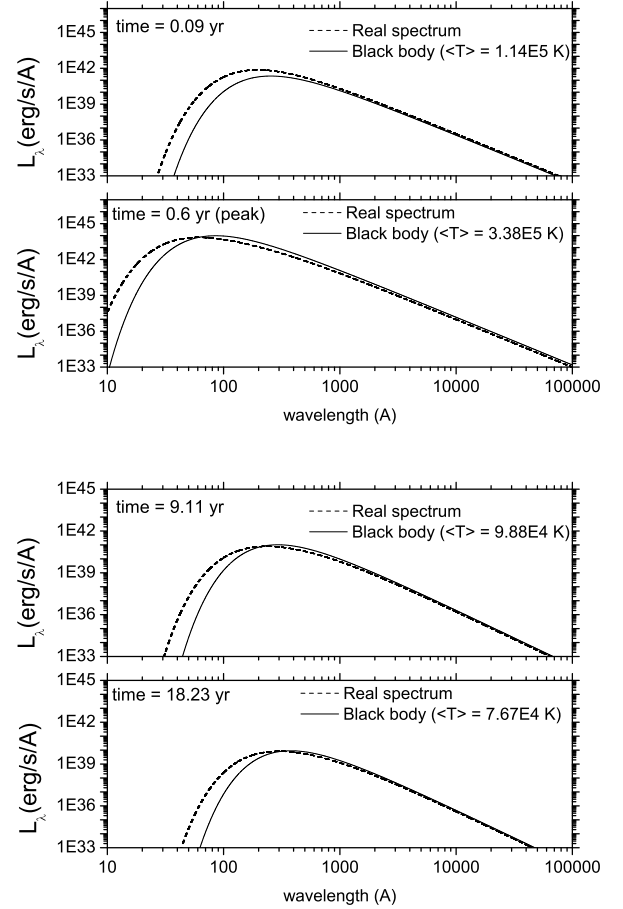


Fig. 9.— Snapshots comparing “real” (dashed curve) and equivalent blackbody spectra (solid curve) for model A3. The temperature charactering the blackbody spectrum is the average effective temperature calculated as explained in the text. The upper panel correspond to a phase in which the disk is still in formation while the first middle panel shows the spectrum at the luminosity peak. Late evolutionary stages are displayed in the remaining two panels.

factors of two and six than the theoretical average effective temperature but such a difference was expected since the disk does not radiate like a blackbody as we have already mentioned.

In spite of the fact that the considered model was not optimized, it gives a reasonable description of the existing data on NGC 3599, consistent with a tidal disruption event involving a solar mass star and a BH of mass $\sim 10^7 M_\odot$. Note that using the $M_{\text{bh}} - \sigma$ relation, the resulting black hole mass in the center of NGC 3599 is $1.3 \times 10^6 M_\odot$ and using the $M_{\text{bh}} - L_b$ relation the derived mass is $3.5 \times 10^7 M_\odot$ (Esquej et al. 2008).

3.4.2. IC 3599

A highly variable X-ray emission originated from the Sb(pec?) galaxy IC 3599 ($z = 0.0215$) was detected by the *ROSAT Wide Field Camera* during its all sky survey (Brandt, Pounds & Fink 1995). The maximum observed luminosity in soft X-rays exceeded $10^{43} \text{ erg s}^{-1}$ and decayed by two orders of magnitude in a timescale of about one year. Optical spectra taken 6.3 yr after the X-ray outburst indicate the presence of strong [OIII] emission and some high-ionization species like HeII λ 4686 and [FeVII] λ 6078, whose intensities have not changed considerably with respect to data obtained just after the event (Komossa & Bade 1999). The possibility that such an outburst could be associated with a tidal disruption event was discussed by Grupe et al. (1995) and Komossa & Bade (1999). In Figure 10 (lower panel) we have plotted the soft X-ray luminosity curve derived from model A2 and *ROSAT* data for IC 3599 taken from Grupe et al. (1995). Again, we emphasize that the considered model gives a good representation of these data but parameters were not optimized in order to obtain a “best fit”. If correct, the model indicates an SMBH in the center of IC 3599 with a mass around $10^7 M_\odot$ and that the disrupted star had a mass close to the solar value. Comparison between the theoretical X-ray light curve and data also indicates that detection occurred close to the luminosity peak (~ 20 days after) and about 5 months after the beginning of the circularization process. The mean effective temperatures derived from model A2 corresponding to the considered data points are respectively 26.6 eV, 15.9 eV, 15.8 eV and 14.4 eV. These should be compared with the values given by Grupe et al. (1995), i.e., 94 eV, 65 eV, 80 eV and 62 eV. These values are, on the average, a factor of four higher than those derived from model A2, confirming the trend already discussed.

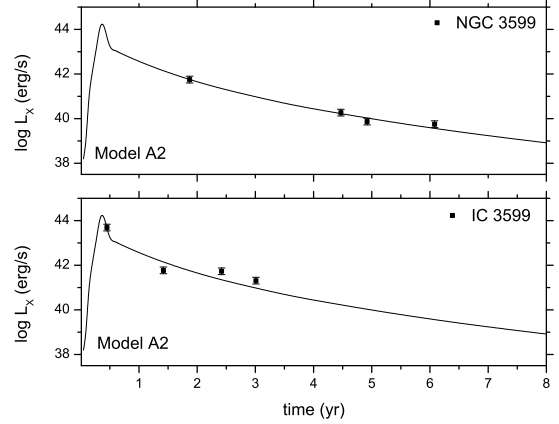


Fig. 10.— Upper panel: evolution of the X-ray luminosity in the 0.2-2.0 keV range for model A2 (solid curve) compared with data on NGC 3599. The first two observations are from *XMM*, the third from *Swift* and the last from *Chandra*. Lower panel: evolution of the X-ray luminosity derived also from model A2 and data on IC3599 by *ROSAT*.

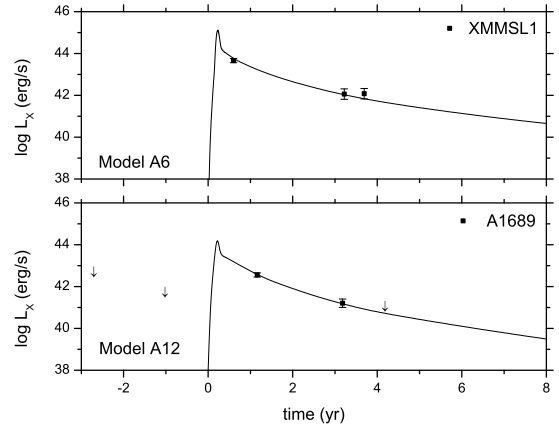


Fig. 11.— Upper panel: evolution of X-ray luminosity in the 0.2-2.0 keV range for model A6 (solid curve) compared with data on XMMSL1. The first two observations are from *XMM-Newton* and the third from *Swift*. Lower panel: evolution of the X-ray luminosity derived from model A12 and data on the outburst detected in the cluster A1689 by *Chandra* and *XMM-Newton*. Small arrows indicate upper limits.

3.4.3. XMMSL1

The X-ray emission of XMMSL1 (or SDSS J132342.3+482701) was detected in 2003.92 by *XMM-Newton* (Esquej et al. 2008). The variable X-ray emission is originates from a non-active galaxy whose spectrum displays only absorption features. This galaxy is at redshift $z = 0.0875$, and its stellar velocity dispersion is $80 \pm 10 \text{ km s}^{-1}$ (Esquej et al. 2008). Using the $M_{\text{bh}} - \sigma$ relation, the measured velocity dispersion implies that the central black hole has a mass around $2 \times 10^6 M_{\odot}$. Model A6 is defined by a black hole of similar mass ($3 \times 10^6 M_{\odot}$) and a disrupted star of one solar mass. In Figure 11 (upper panel), the X-ray luminosity curve derived from this model is shown in comparison with data from *XMM-Newton* and *Swift*. The model is consistent with observations, but the small number of data points avoids a more robust conclusion. If correct, the model indicates that detection occurred 5-6 months after the X-ray peak luminosity.

X-ray data at the high-state emission if fitted by a blackbody spectrum indicate temperatures in the range 33-49 eV (Esquej et al. 2008) while we have derived from model A6 an average effective temperature of 30 eV.

3.4.4. A1689

Chandra observations in 2004.14 detected a variable X-ray emission associated with the galaxy SDSS J131122.15-012345.6, a member of the cluster A1689 at a redshift of $z = 0.183$ (Maksym, Ulmer & Eracleous 2010). Prior observations performed either in 2000.29 and in 2001.98 by *Chandra* and *XMM-Newton* have not detected the source. Since only two data points and three upper limits are available, it is difficult to constrain a model. The X-ray light curve derived from model A12, characterized by a $10^7 M_{\odot}$ and a disrupted star of half solar mass gives an acceptable representation of the existing data, satisfying also the pre-outburst constraints (see Figure 11, lower panel).

In its high-state emission Maksym, Ulmer & Eracleous (2010) derived from their fits an equivalent blackbody temperature of 120 eV whereas the average mean temperature derived from model A12 is 21.6 eV. This is again consistent with our previous statement that blackbody fits to X-ray data overestimates the true mean temperature of the disk.

3.4.5. NGC 5905

NGC 5905 is a SB(r)b galaxy located at $z = 0.0113$. A variable X-ray emission from this object was detected in the all sky survey by *ROSAT* around 1990 July (Bade, Komossa & Dahlem 1996). Subsequent observations in

the following six days indicated an increase in the flux by a factor of three and pointed observations performed in 1993 showed that the X-ray emission decayed by almost two orders of magnitude, suggesting the occurrence of an outburst event. Its optical spectrum is dominated by strong absorption lines, and narrow forbidden lines ([OI], [OIII], [NII], and [SII]), as well as Balmer emission lines (Komossa & Bade 1999). Since no particular signals of nuclear activity are seen, the galaxy may have a “dormant” central SMBH that could be responsible for a tidal disruption event associated with the X-ray outburst detected by *ROSAT*.

In order to analyze the existing X-ray data on this event, we recall that the rising part of the light curve in our models is determined by the accretion (or viscous) timescale, which is typically of the order of 4 months. If the observed increase in the X-ray flux seen in the early detection by *ROSAT* corresponds to the raising part of the flare, implying a timescale of ~ 5 days, then none of our models are able to fit the X-ray light curve. Such a short raising timescale implies a very small disk that could be formed if the disrupted object was a low-mass star. For instance, a M8V star with a mass of $\sim 0.1 M_{\odot}$ and a radius of $\sim 0.12 R_{\odot}$ will correspond, using Equation 7, to an accretion timescale of about 10 days for a critical Reynolds number equal to 500. The analysis by Li, Narayan & Menou (2002) leads to a similar conclusion.

4. Conclusions

In the present paper we report the results derived from numerical solutions of the hydrodynamic equations describing the evolution of an accretion disk originated from the debris resulting from a tidal disruption event. Models are characterized by the central black hole mass, the properties of the disrupted star (mass and radius) and the critical Reynolds number defining the onset of a turbulent flow.

Once the bound material begins to circularize, there is a finite amount of time before the black hole begins to accrete mass, which is defined by the accretion (or “viscous”) timescale t_a . In the beginning of the circularization process a torus-like structure is formed close to the periape level (assumed here to coincide with the tidal radius), expanding inward until the last stable orbit is reached in a timescale $\sim t_a$. This initial phase, when the disk is still being formed, corresponds to the rising part of the light curve that characterizes the event. The peak luminosity marks the beginning of the accretion process by the central black hole, consequently also occurring in a timescale t_a . The presence or lack of a

“bump” in the light curve after the luminosity peak depends on the ratio t_a/t_{\max} or, in other words, how the accretion timescale compares with the instant t_{\max} where the maximum of the fallback rate occurs. “Bumps” occur when $t_a/t_{\max} < 1$, corresponding to models of series “M”. They are absent in models of series “A”, satisfying the opposite condition. The comparison with actual observations leads to the conclusion that “A”-models are more adequate to represent the existing data.

Peak bolometric luminosities for “A”-models are in the range $10^{45} - 10^{46}$ erg s⁻¹ while peak luminosities in soft X-rays (0.2-2.0 keV) are one order of magnitude lower. For the same parameters, models of series “M” have peak luminosities about one order of magnitude lower. The characteristic timescale for the luminosity to decay by two orders of magnitude after the peak is about 3.3 yr for models “A” and 43.3 yr for models “M”. These values are considerably smaller than the timescales derived by Cannizzo, Lee & Goodman (1990) from their models.

The effective temperature varies along the disk surface as well as in time. The maximum values are observed when the debris material reaches the last stable orbit for the first time, which is of the order of $(1 - 2) \times 10^6$ K. At this particular moment, the most important temperature gradient across the disk surface is observed. In the last evolutionary phases, the temperature distribution along the disk surface becomes more uniform and the disk becomes more and more “cold” as the source of debris is exhausted.

The resulting spectrum is not that of a blackbody since it results from the integrated emission from a surface having a temperature distribution and, despite the fact that the effective optical depth is much larger than unity, scattering by free electrons and free-free absorption contributes equally to the opacity and, consequently, the local spectrum differs from a blackbody. Comparison between true spectra and those computed with a blackbody characterized by an average temperature giving the same total disk emission, indicates that they agree in the Rayleigh-Jeans region but that the blackbody underestimates the true flux at short wavelengths. Consequently, blackbody fits of X-ray data tend to overestimate the true mean disk temperature. In fact, the temperature derived from the blackbody fits should be identified with the color X-ray temperature rather than the true average disk effective temperature.

In spite of the fact that the present models were not computed to explain any particular observation, they were tested for several tidal flare event candidates. X-ray data on NGC 3599 and IC 3599 can be satisfactorily represented by model A2, which involves a black hole of

$10^7 M_{\odot}$ and a disrupted star of one solar mass. According to our model, NGC 3599 was first detected ~ 1.5 yr after the X-ray luminosity peak and IC 3599 very close to it (~ 20 days after). The soft X-ray light curve derived from Model A6, characterized by a $3 \times 10^6 M_{\odot}$ black hole and a disrupted star of one solar mass fits quite well data on XMMSL1. The X-ray peak emission for this object occurred probably 5-6 months before detection at a luminosity of about 10^{45} erg s⁻¹. The outburst that occurred in the cluster A1689 is well represented by model A12 involving a $10^7 M_{\odot}$ black hole but the disrupted star is a low-mass object with half solar mass. Its detection occurred about one year after the peak luminosity according to our model. Finally, none of our models were able to fit the existing data on NGC 5905. The short rise time of the flare points to the disruption and capture of the debris of a low-mass object, and this problem is presently under investigation.

4.1. Acknowledgments

M. M.A acknowledges the Comisión Nacional de Investigación Científica y Tecnológica (CONICYT) de Chile for the fellowship that permitted his stay at the Observatoire de la Côte d’Azur. The authors thank the referee for his/her suggestions that have improved the presentation of this paper.

REFERENCES

- Ayal, S., Livio, M. & Piran, T., 2000, ApJ 545, 772
- Bade, N., Komossa, S. & Dahlem, M., 1996, A&A 309, L35
- Begelman, M.C., 1978, MNRAS 184, 53
- Bicknell, G.V. & Gingold, R.A., 1983, ApJ 273, 749
- Brandt, W.N., Pounds, K.A. & Fink, H.H., 1997, MNRAS 273, L47
- Cannizzo, J.K., Lee, H.M. & Goodman J., 1990, ApJ 351, 38
- Cappelluti, N., Ajello, M., Rebusco, P., Komossa, S., Bongiorno, A., Clemens, C., Salvato, M., Esquej, P., Aldcroft, T., Greiner, J. & Quintana, H., 2009, A&A 495, L9
- Carter, B. & Luminet, J.-P., 1983, A&A 121, 97
- Chambers, K.C., 2007, BAAS 38, 995
- de Freitas Pacheco, J.A. & Steiner, J.E., 1976, ApSS 39, 487
- Donley, J.L., Brandt, W.N., Eracleous, M. & Boller, Th., 2002, AJ 124, 1308

- Esquej, P., Saxton, R.D., Freyberg, M.J., Read, A.M., Altieri, B., Sánchez-Portal, M. & Hasinger, G., 2007, A&A 462, L49
- Esquej, P., Saxton, R.D., Komossa, S., Read, A.M., Freyberg, M.J., Hasinger, G., García-Hernández, D.A., Lu, H., Rodríguez Zaurín, J., Sánchez-Portal, M. & Zhou, H., 2008, A&A 489, 543
- Evans, C.R. & Kochanek, C.S., 1989, ApJ 346, L13
- Frank, J., 1978, MNRAS 184, 87
- Gezari, S., Martin, D.C., Milliard, B. et al., 2006, ApJ 653, L25
- Gezari, S., Heckman, T., Cenko, S.B. et al., 2009, ApJ 698, 1367
- Grupe, D., Beuermann, K., Mannheim, K., Bade, N., Thomas, H.-C., de Martino, D. & Schwobe, A., 1995, A&A 299, L5
- Heinzeller, D. & Duschl, W.J., 2007, MNRAS 374, 1146
- Komossa, S. & Bade, N., 1999, A&A 343, 775
- Khokhlov, A., Novikov, I.D. & Pethick, C.J., 1993, ApJ 418, 181
- Lacy, J.H., Townes, C.H. & Hollenback, D.J., 1982, ApJ 262, 120
- Laguna, P., Miller, W.A., Zurek, W.H. & Davies, M.B., 1993, ApJ 410, L83
- Law, N.M., Kulkarni, S.R., Dekany, R.G. et al., 2009, PASP 121, 1395
- Li, L.-X., Narayan, R. & Menou, K., 2002, ApJ 576, 753
- Lodato, G., King, A.R. & Pringle, J.E., 2009, MNRAS 392, 332
- Maksym, P., Ulmer, M.P. & Eracleous, M., 2010, ApJ 722, 1035
- Montesinos Armijo, M.A. & de Freitas Pacheco, J.A., 2011, A&A 526, 146 (MP11)
- Nolthenius, R.A. & Katz, J.I., 1982, ApJ 263, 377
- Oshuga, K., Mineshige, S., Mori, M. & Umemura, M., 2002, ApJ 574, 315
- Paczynski, B. & Wiita, P.J., 1980, A&A 88, 23
- Pichon, B., 1985, A&A 145, 387
- Piran, T., 1978, ApJ 221, 652
- Press, W.H. & Teukolsky, S.A., 1977, ApJ 213, 183
- Rees, M.J., 1988, Nature 333, 523
- Shakura, N.I. & Sunyaev, R.A., 1973, A&A 24, 337
- Strubbe, L.E. & Quataert, F., 2009, MNRAS 400, 2070
- Ulmer, A., 1999, ApJ 514, 180
- van Velzen, S., Farrar, G.R., Gezari, S., Morrel, N., Zaritsky, D., Ostman, L., Smith, M. & Gelfano, J., 2010, arXiv:1009.1627